A Side-Arm-Switched Directional Coupler as a Single-Step Attenuator of High Long-Term Stability

P. I. SOMLO, SENIOR MEMBER, IEEE, AND I. G. MORGAN

Abstract—This paper describes a single-step attenuator for values below 0.1 dB of very high long-term stability. This uses a directional coupler with a fixed short circuit at one coupled port and a switchable match-or-short at the other. The value of the attenuation step in the transmitting path is governed by the coupling and thus very small steps may be achieved readily. The attenuator is analyzed in full and relationships for a leaky directional coupler are given, together with a flowgraph model of a nonideal coupler.

I. INTRODUCTION

At a meeting of the CIPM—Advisory Committee on Electricity, Radio Frequency Working Group held in 1972 at Sèvres, France, a number of standards laboratories[1] agreed to participate in an international intercomparison of the measurement of "small" microwave attenuations at 10 GHz. This laboratory undertook the development of a 0.01-dB step attenuator to be used as a transfer standard in this intercomparison. Subsequently, a 0.001-dB step was also constructed for this purpose.

The aim of having high long-term stability, flat frequency response at the operating frequency, small reflections, and positive switching action defining the two states is achieved with a directional coupler, terminated at one of the coupled ports in a full reflection and switched at the other coupled port between a matched load and a full reflection. The attenuation step in either state of the switch is mainly governed by the coupling geometry, an inherently stable configuration for waveguide couplers.

II. DISCUSSION

A. *Principle of Operation*

Consider first an ideal coupler operating between matched ports and with one coupled port terminated in a full reflection $\epsilon^{j\phi'}$. The change in transmission when the other coupled port is switched between a full reflection, $s = \epsilon^{j\phi''}$, and a matched load, $s = 0$, will be found. The schematic circuit and the flowgraph of this ideal device are shown in Fig. 1(a).

The transmission of the ideal coupler is $\boldsymbol{T} = T\epsilon^{j\alpha}$ and for a loss-free coupler (see the Appendix) the coupling is $\boldsymbol{C} = C\epsilon^{j\beta} = jC\epsilon^{j\alpha}$. The transmission from port 1 to port 2 is:

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The authors are with the National Measurement Laboratory, CSIRO, Sydney, N. S. W., Australia.
[1] IEN (Italy), NOM (Hungary), NSL (Australia), PTB (Germany) (pilot laboratory), RIND (Sweden), RRE (United Kingdom).

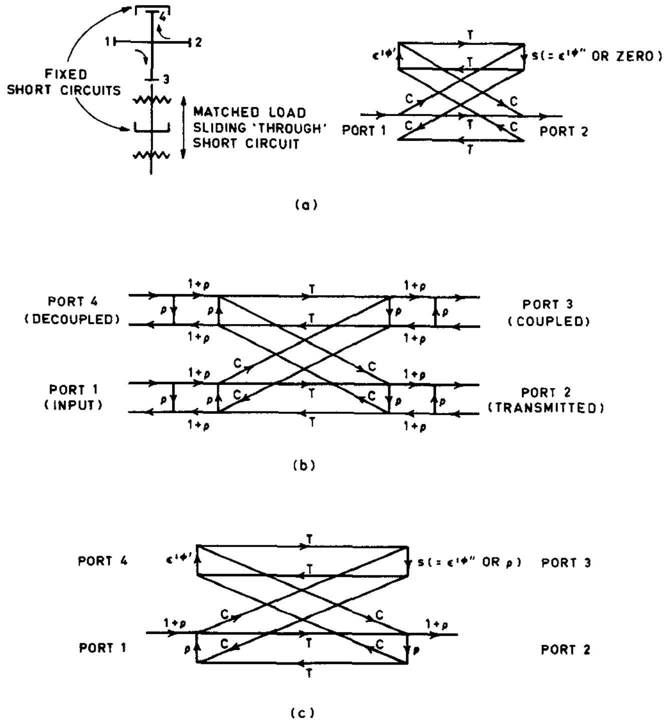


Fig. 1. (a) Schematic circuit and flowgraph of step attenuator. (b) Flowgraph of practical (leaky) coupler. The cause of leakage and reflection are the reflections introduced at the four ports of an ideal coupler. (c) Relevant branches of a practical step attenuator flowgraph for the calculation of the transmission from port 1 to port 2. Port 3 is terminated in a permanent full-reflection and port 4 is terminated in a short-to-match switch.

$$S_{21} = \frac{\boldsymbol{T}(1 - \boldsymbol{T}^2\epsilon^{j\phi'}s) + \boldsymbol{C}^2\boldsymbol{T}\epsilon^{j\phi'}s}{1 - \boldsymbol{T}^2\epsilon^{j\phi'}s} = \frac{T\epsilon^{j\alpha}(1 - s\epsilon^{j(2\alpha+\phi')})}{1 - T^2s\epsilon^{j(2\alpha+\phi')}}. \tag{1}$$

Considering now the two states of the switch, the state being indicated by a superscript corresponding to the magnitude of s, and substituting in (1) for $s = 0$ we obtain:

$$S_{21}^{(0)} = T\epsilon^{j\alpha}. \tag{2}$$

If s is in the other state, $s = \epsilon^{j\phi''}$, from (1)

$$S_{21}^{(1)} = \frac{T\epsilon^{j\alpha}(1 - \epsilon^{j(2\alpha+\phi'+\phi'')})}{1 - T^2\epsilon^{j(2\alpha+\phi'+\phi'')}}. \tag{3}$$

It is seen from (3) that if the side arm is made resonant between the two full reflections, $2\alpha + \phi' + \phi'' = 2n\pi$ where $n = 0,1,2,\cdots$, $S_{21}^{(1)} = 0$, then the power flow is cut off completely between ports 1 and 2. However, for this application, the coupled arm is made *antiresonant*, i.e., $2\alpha + \phi' + \phi'' = n\pi$ where $n = 1,3,5,\cdots$, then

$$S_{21}^{(1)} = \frac{2T\epsilon^{j\alpha}}{1+T^2}. \tag{4}$$

The ratio between the transmissions for the two states of the switch for the ideal coupler is then

$$R_i = \frac{S_{21}^{(1)}}{S_{21}^{(0)}} = \frac{2}{1+T^2} = \frac{2}{2-C^2}. \tag{5}$$

Thus the magnitude of the step is a function only of the coupling, and since R_i is real, a phase-shift free step has been obtained. Furthermore, since (5) is independent of ϕ' and ϕ'', the position of the antiresonant section relative to the coupling mechanism is irrelevant.

The reflections of the step attenuator in the two states of the switch $s = 0$ and $s = \epsilon^{j\phi''}$ for the antiresonant condition are

$$S_{11}^{(0)} = 0$$

$$S_{22}^{(0)} = C^2\epsilon^{j\phi'}$$

$$S_{11}^{(1)} = \frac{-C^2\epsilon^{j(2\alpha+\phi'')}}{1 - T^2\epsilon^{j(2\alpha+\phi'+\phi'')}} = \frac{-C^2\epsilon^{j(2\alpha+\phi'')}}{1+T^2}$$

$$= \frac{-C^2\epsilon^{j(2\alpha+\phi'')}}{2-C^2}. \tag{6}$$

Thus the reflections as shown by (6) are in the order of C^2 and $C^2/2$, respectively, and for coupling coefficients of -20 dB or less, in either state of the switch the reflection coefficients at the two ports are less than 0.01 in magnitude. The performance of an ideal directional coupler as a step attenuator is summarized in Fig. 2, and Fig. 3 shows qualitatively the transmission of a typical coupler in the two states of the switch. Each "notch" of the "comb" indicates a resonance in the coupled arm.[2]

B. Perturbations

1) Departure from Antiresonance: Let $2\alpha + \phi' + \phi'' - \delta = n\pi (n = 1,3,\cdots)$ where δ is the excess phase shift by which the coupled arm is detuned from antiresonance. Substituting in (3) and with $S_{21}^{(0)}$ remaining as given by (2), the step ratio now is

$$R = \frac{S_{21}^{(1)}}{S_{21}^{(0)}} = \frac{1+\epsilon^{-j\delta}}{1+T^2\epsilon^{-j\delta}} = \frac{1+\epsilon^{-j\delta}}{1+(1-C^2)\epsilon^{-j\delta}}$$

$$= \frac{1}{1 - \dfrac{\epsilon^{-j\delta}}{1+\epsilon^{-j\delta}}C^2}. \tag{7}$$

[2] This plot was obtained by a computerized reduction of a realistic model for a 20-dB coupler, with 20-dB directivity, and insertion loss of about 0.003 dB. The method is based on [1], is written in BASIC, and was used throughout this work for numerical checks. This general-purpose flowgraph reduction program is available on request. (Note that there is a misprint in [1]. On p. 753 the fifth line should read instead: "... times entry γ_{ij} times row j to row i, where ....")

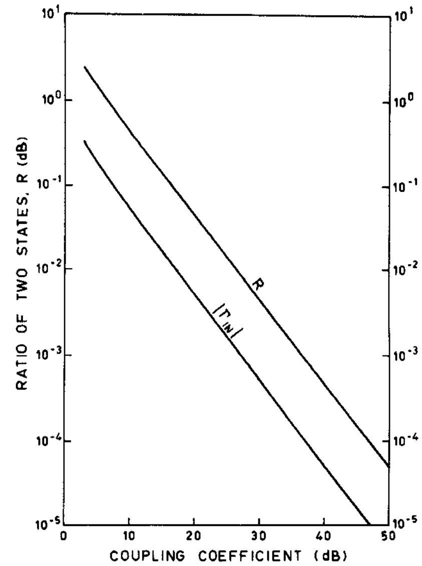


Fig. 2. Theoretical attenuation step value and input reflection for an ideal coupler, as a function of the coupling coefficient.

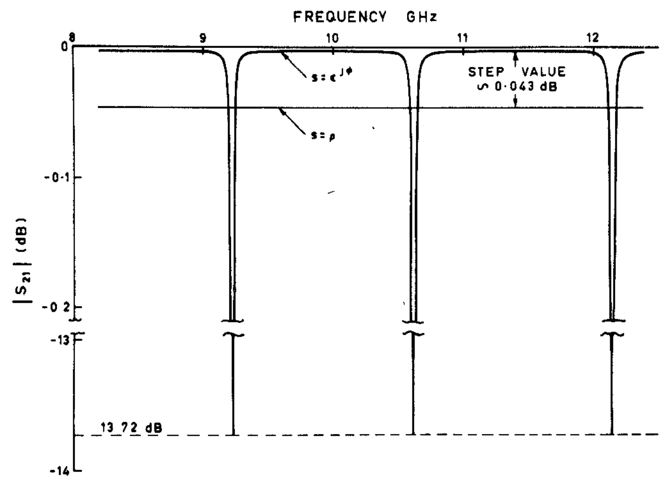


Fig. 3. Resulting transmission of computerized flowgraph reduction. Details of model: coupling and directivity (constant); 20-dB separation of two short circuits by WR 90 waveguide; 90-mm waveguide attenuation; 0.003 dB/cm. The "notches" indicate a resonant condition in the coupled arm; the design frequency is (quasi) halfway between, at the antiresonant condition. Both states of the switch are shown.

It is seen that if $\delta = 0$ (antiresonant condition), (7) is equal to (5); however, for $\delta \neq 0$, R is perturbed. Let the perturbing factor in (7) be k, then

$$k = \frac{\epsilon^{-j\delta}}{1+\epsilon^{-j\delta}} = \frac{\cos\delta - j\sin\delta}{1+\cos\delta - j\sin\delta} = \frac{1+\cos\delta - j\sin\delta}{2(1+\cos\delta)}$$

$$= \frac{\epsilon^{-j(\delta/2)}}{(2+2\cos\delta)^{1/2}}. \tag{8}$$

The locus of k may be shown to lie on the line Re $(k) = 0.5$ and for a detuning of δ its angle is $\delta/2$ as shown by (8). From (7) and (8) the perturbed step ratio may be rewritten:

$$R = \frac{1}{1 - C^2/2 + j(C^2/2)\tan\delta/2}. \tag{9}$$

Comparing the perturbed step (9) with the ideal step (5) we find that the detuning affects R in quadrature; thus for small δ the magnitude of R is only slightly affected. As a numerical example, for a 20-dB coupler, as shown by Fig. 2, the theoretical step ratio is about 0.0435 dB and this figure is changed by 0.001 dB for $\delta \approx 140°$, indicating the relative insensitivity of the step value to the exactness of the antiresonant condition of the coupled arm.

2) Finite Directivity: For simplicity, twofold symmetry of the model of a coupler is assumed, as shown in the Appendix. The transmission for the model is

$$S_{21} = \frac{(1+\rho)^2 T\epsilon^{j\alpha}(1 - s\epsilon^{j(2\alpha+\phi')})}{1 + s(-T^2\epsilon^{j(2\alpha+\phi')} + \rho\epsilon^{j2\alpha} - \rho T^2\epsilon^{j2\alpha} + \rho^2\epsilon^{j(4\alpha+\phi')}) - \rho^2T^2\epsilon^{j2\alpha} + \rho\epsilon^{j(2\alpha+\phi')} - \rho T^2\epsilon^{j(2\alpha+\phi')}}. \tag{10}$$

Setting s to a full reflection, $s = \epsilon^{j\phi''}$, and maintaining antiresonance in the coupled arm $2\alpha + \phi' + \phi'' = n\pi (n = \text{odd})$, $S_{21}^{(1)}$ is obtained. Similarly, switching the termination to a matched load beyond the shunt susceptance thereby making $s = \rho$, $S_{21}^{(\rho)}$ is obtained. For moderate reflections and not too tight coupling $\rho^2 \ll 1$ and $C^2 \ll 1$, i.e., $T^2 \approx 1$ and after neglecting terms cubic in ρ the step ratio will be

$$R = \frac{S_{21}^{(1)}}{S_{21}^{(\rho)}} \approx \frac{2}{1+T^2} \pm \rho C^2. \tag{11}$$

The first term in (11) is the theoretical step ratio for an ideal coupler as given in (5) and is perturbed by the presence of ρ. The directivity D (from the Appendix) is

$$D = \frac{S_{31}}{S_{41}} = \frac{1 + \rho^2\epsilon^{j2\alpha}}{2\rho T\epsilon^{j\alpha}} \tag{12}$$

and for the case where the above inequalities and approximations apply,

$$|D| \approx \frac{1}{2|\rho|}$$

or in decibels

$$|D|^{(\text{dB})} \approx 20 \log_{10} \frac{1}{2|\rho|}. \tag{13}$$

Thus for 20-dB directivity $|\rho| = 0.05$ and for 40-dB directivity $|\rho| = 0.005$. Substituting these figures in (11) and using the link between T and C we find that for $D = 20$ dB the step ratio (in decibels) is perturbed by 10 percent and for $D = 40$ dB it is perturbed by 1 percent.

III. SPECIAL APPLICATIONS

A. Phase-Shift-Free Level Trimmer

The operation resembles somewhat that of Beatty and Fentress' divider circuit [2], but here only one coupler is needed and in one state the theoretical insertion loss is very close to zero. As shown in Section II-A, for an ideal coupler with the side arm at antiresonance or one coupled port terminated in a matched load, there will be only an amplitude change of the transmitted wave and no phase shift between the two states. If the transition between the two states is made gradual, a variable attenuator results. With an ideal attenuator in front of one of the short circuits, a gradual change in s may be made in Fig. 1(a), and writing $s = k\epsilon^{j\phi''}$ and substituting into (1) we obtain

$$S_{21}^{(k)} = T\epsilon^{j\alpha} \frac{1+k}{1+(1-C^2)k} \tag{14}$$

remembering that now k is the attenuation expressed as a power ratio. Normalizing to the maximum step available for a given coupling in decibels, the attenuation law as a function of k will be

$$R^{(k)} = \frac{20 \log_{10} \{(1+k)/[1+(1-C^2)k]\}}{20 \log_{10} [2/(2-C^2)]}. \tag{15}$$

A plot of this law is given in Fig. 4(a). For example, using a 20-dB coupler the maximum step available from Fig. 2 is about 0.043 dB, and with an attenuator set to 3 dB in front of one of the shorts in the coupled arm, from Fig. 4(a), 0.667 of 0.043 dB = 0.029 dB will be obtained.

B. Quasi-Constant-Amplitude Phase Trimmer

As shown in Section II-B1, the step value is very insensitive to departure from antiresonance in the coupled arm, and this property is utilized here. Sliding one of the two shorts in the coupled arm, from (9), on varying the electrical length between the two shorts by δ (from the antiresonant condition), the phase shift will be

$$\gamma = \frac{1}{C^2} \arctan\left(\frac{(C^2/2)\tan\delta/2}{1 - C^2/2}\right). \tag{16}$$

A plot of this phase shift divided by C^2 is shown in Fig. 4(b). For example, for a 20-dB coupler if one of the shorts is moved by 30° from the antiresonant condition, the resultant phase shift in the transmission (with virtually no change in amplitude) will be about 18°/100 = 0.18°.

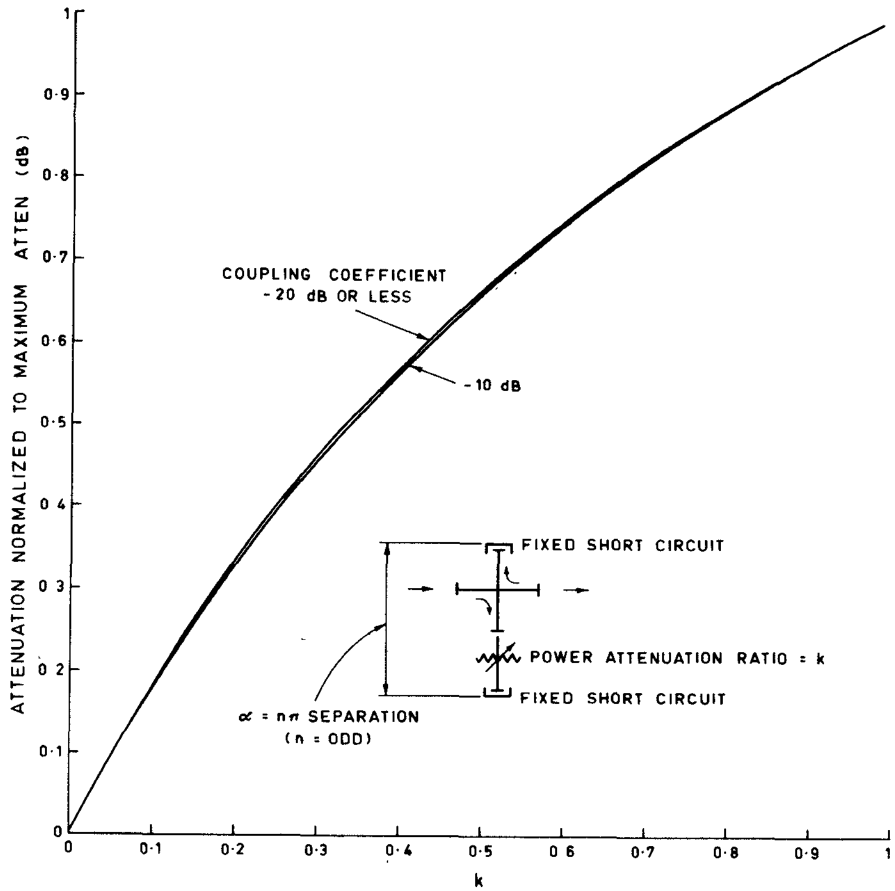
C. Stable "Micromodulator"

In some applications it may be desired to "tag" a microwave signal by a very small audio modulation of a very stable modulation depth. In this case s may be realized as a p-i-n diode approximating a full reflection and a matched load in its two states. The amount of modulation for a given coupler may be read off directly from Fig. 2. The fact that this very stable modulation is achieved with a modulator of almost zero insertion loss may render this circuit suitable for calibrating the overall incremental sensitivity of radiometers using the input signal itself.

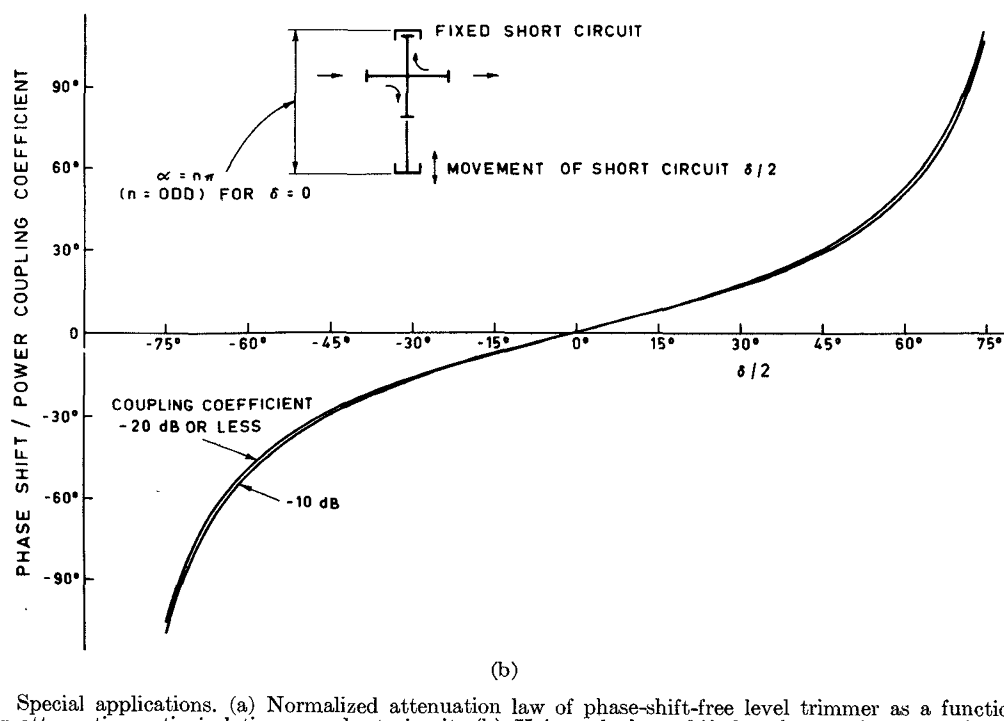
IV. DESIGN CONSIDERATIONS

A. The Match-to-Short Switch

For the present application the utmost stability was sought; therefore, the obvious solution of converting a matched load to a full reflection by placing a commercially



(a)



(b)

Fig. 4. Special applications. (a) Normalized attenuation law of phase-shift-free level trimmer as a function of a power attenuation ratio isolating one short circuit. (b) Universal phase-shift law for quasi-constant-level phase trimmer, as a function of the movement of a short circuit, detuning the coupled arm from the antiresonant condition.

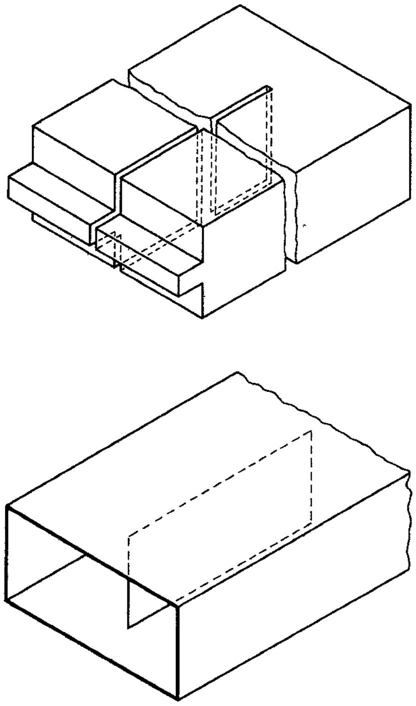


Fig. 5. Exploded view of the short-to-match switch. The slotted tuned matched termination has two positions in the bifurcated waveguide. When the load is sufficiently withdrawn, the divided guide is below cutoff and a full-reflection results. When it is brought forward, a matched condition is provided, the lossy body of the termination isolating the cutoff waveguide. The load slides on Teflon spacers (not shown).

available waveguide shorting switch in front of the load was rejected in favor of a solution that is simple, yet avoids metallic sliding contacts.

The full reflection was achieved by dividing the waveguide by a thin central metallic partition parallel to the electric field. Since the two halves of the waveguide are below cutoff, a full reflection results. The conversion of this "short circuit" into a match was achieved by slotting the center of a matched load, permitting it to be withdrawn so far into the cutoff region, or to be pushed sufficiently ahead of the cutoff region so that the "short" is isolated from the leading edge of the load by more than 50 dB. An exploded view of the essential components of this switch is shown in Fig. 5.

B. The Directional Coupler

Since the separation of the "notches" in Fig. 3 (i.e., resonances in the coupled arm) is a function of the separation of the two short circuits, a cross-guide coupler instead of a longer multihole coupler was chosen. As may be read from Fig. 2, for the desired attenuation steps of 0.01 and 0.001 dB the coupling coefficients required are about 26.6 and 36.6 dB, respectively.

V. CONCLUSION

It has been shown that the transmission in the through arm of a directional coupler may be changed in a calculable way by having one coupled port terminated in a short circuit, and switching the termination of the other coupled arm from a zero- to a full-reflection. This level change is virtually free from phase change. The value of the attenuation step is primarily a function of the coupling coefficient of the coupler, which is inherently stable, and since the switching that takes place is physically remote from the coupling region, a very stable long-term performance results.

Applications of this side-arm-switched directional coupler include: 1) a secondary standard for small attenuation; 2) a zero insertion-loss sensitivity calibrator (for radiometers, for instance); 3) a constant phase level trimmer; and 4) a quasi-constant-level phase trimmer. A computerized flowgraph reduction program is offered on request.

APPENDIX

Any loss-free directional coupler can be represented as an ideal directional coupler with loss-free two-ports connected to at least three of its ports [3]. For simplicity, this appendix treats only couplers with twofold symmetry, as none of the conclusions in the paper are invalidated by this restriction. Any practical loss-free coupler with this symmetry can therefore be regarded as an ideal core with transmission to the "through" arm of $\boldsymbol{T} = T\epsilon^{j\alpha}$ and having a normalized shunt susceptance B at each of its ports. As has been shown [3], the transmission to the coupled arm of the ideal core is $\boldsymbol{C} = \pm jC\epsilon^{j\alpha}$ where $T^2 + C^2 = 1$, from the conservation of energy. The indeterminate sign is taken as positive, without affecting the conclusions. For a given magnitude of B the amplitude and phase of $\rho = -jB/(2 - jB)$ are determined. For complete generality in this symmetrical case it is necessary to add equal transmission-line lengths between the locations of the susceptances and the actual physical ports. However, this is only of importance in external measurements to derive the parameters of the model. For these reasons the flowgraph representation of a practical directional coupler is that shown in Fig. 1(b).

In interference-type directional couplers, which depend on the separation of the planes of discrete coupling by $\lambda_g/4$ for high directivity at the design frequency, α will be approximately $\pi/2$. Thus the input reflection with matched loads at the other ports will be small in comparison with ρ. In this case an estimate of ρ may be made either by tuning the transmission arm or the coupled arm for zero leakage and measuring the input reflection coefficient, or by measuring the directivity.

For the model of Fig. 1(b) the scattering parameters of a practical coupler may be expressed in terms of those of the ideal core and of the perturbing reflections as follows:

reflection:

$$\begin{aligned} S_{11} &= \rho + \rho(1+\rho)^2(\boldsymbol{T}^2 + \boldsymbol{C}^2 - \rho^2(\boldsymbol{T}^2 - \boldsymbol{C}^2)^2)/d \\ &= \rho + \rho(1+\rho)^2(1 - 2C^2 - \rho^2\epsilon^{j2\alpha})\epsilon^{j2\alpha}/d \end{aligned}$$

transmission:

$$\begin{aligned} S_{21} &= \boldsymbol{T}(1+\rho)^2(1 - \rho^2(\boldsymbol{T}^2 - \boldsymbol{C}^2))/d \\ &= T(1+\rho)^2(1 - \rho^2\epsilon^{j2\alpha})\epsilon^{j\alpha}/d \end{aligned}$$

coupling:

$$S_{31} = \boldsymbol{C}(1+\rho)^2(1+\rho^2(\boldsymbol{T}^2 - \boldsymbol{C}^2))/d = jC(1+\rho)^2(1+\rho^2\epsilon^{j2\alpha})\epsilon^{j\alpha}/d$$

leakage:

$$S_{41} = 2\rho\boldsymbol{CT}(1+\rho)^2/d = j2\rho CT(1+\rho)^2\epsilon^{j2\alpha}/d$$

where

$$d = 1 - 2\rho^2(\boldsymbol{T}^2 + \boldsymbol{C}^2) + \rho^4(\boldsymbol{T}^2 - \boldsymbol{C}^2)^2 = 1 - 2\rho^2(1 - 2C^2)\epsilon^{j2\alpha} + \rho^4\epsilon^{j4\alpha}.$$

It is seen from these equations that if $\rho = 0$ the parameters of an ideal coupler are returned with $S_{11} = S_{41} = 0$, $S_{21} = \boldsymbol{T}$, and $S_{31} = \boldsymbol{C}$. Furthermore, interchanging $\boldsymbol{T}$ and $\boldsymbol{C}$ is seen to interchange S_{21} and S_{31}. The model also shows that for loose coupling and small imperfections, i.e., $C^2 \ll 1$ and $\rho^2 \ll 1$, if the coupling region is a quarter-wavelength long giving $\alpha \approx \pi/2$ and $\boldsymbol{T}^2 \approx -1$, then the input reflection $S_{11} \approx 0$, as mentioned previously for interference-type couplers.

This demonstrates that the simple flowgraph model of a practical directional coupler of Fig. 1(b) may be used to derive the mathematical relationships describing the circuit, and the flowgraph may be used as a general-purpose building block to insert directional couplers into larger microwave networks to be analyzed by computerized flowgraph reduction programs (see footnote 2).

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REFERENCES

[1] N. Munro and P. D. McMorran, "Signal-flow-graph reduction, Mason's rule and the system matrix," *Electron. Lett.*, vol. 6, pp. 752–754, Nov. 1970.

[2] R. W. Beatty and G. H. Fentress, "An attenuation and phase shift divider circuit," *Proc. IEEE* (Lett.), vol. 56, pp. 2063–2064, Nov. 1968.

[3] W. K. Kahn and R. L. Kyhl, "The directional coupler core of an arbitrary, lossless, reciprocal 4-port," *Proc. IRE* (Corresp.), vol. 49, pp. 1699–1700, Nov. 1961.

Short Papers

On the Existence Range of the S Parameters of a Passive Two-Port Network

EDOARDO CARLI, MEMBER, IEEE, AND TULLIO CORZANI, MEMBER, IEEE

Abstract—**The existence range of the S parameters of a passive two-port is analyzed and various limitations on their moduli are deduced. In particular, it is shown that when three moduli are given, the last, in general, may be upper and lower bounded by passivity conditions.**

I. INTRODUCTION

In the last three years some considerations on the bounds of the VSWR of a passive two-port network have appeared in the literature [1]–[3]. The authors dealt with the same subject some years ago, investigating the existence range of the S parameters of a passive two-port network. In particular, various limitations on the S parameters were derived. The results were used to obtain some relations between the passivity and absolute stability conditions and to determine the bounds of the input VSWR of a loaded passive two-port [4]. Furthermore, the limits of the nonreciprocity of a passive two-port and the best loss condition for an isolator were analyzed [5].

In this short paper some general limitations for the S parameters of a passive two-port network are deduced in both the nonreciprocal and the reciprocal case.

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The authors are with the Istituto di Elettrotecnica e di Elettronica, Università di Trieste, Trieste, Italy 34127.

In particular, it is shown that when all but one of the parameters are known the modulus of the last parameter can be, in general, upper and lower bounded by passivity conditions.

II. PASSIVITY CONDITIONS

For the sake of simplicity, only the case of lossy two-port networks will be considered; in that case the power entering the network is always positive and the Hermitian form giving this power as a function of the incident waves is definite positive.

The discussion of the general case of a nonnegative definite Hermitian form, including lossless networks and conditionally lossy networks (i.e., networks whose entering power can vanish for some particular excitations), does not modify the conclusions, whereas it requires many involved specifications. This case is discussed in detail in [4].

For a lossy nonreciprocal network it must be

$$1 - |S_{11}|^2 - |S_{21}|^2 > 0 \quad (1)$$

$$(1 - |S_{11}|^2)(1 - |S_{22}|^2) + (1 - |S_{12}|^2)(1 - |S_{21}|^2) - 1 - 2|S_{11}||S_{22}||S_{12}||S_{21}|\cos\alpha > 0 \quad (2)$$

where

$$\alpha = \arg S_{11} + \arg S_{22} - \arg S_{12} - \arg S_{21}. \quad (3)$$

As is well known, conditions (1) and (2) mean, respectively, that the first element and the determinant of the dissipation matrix $(\mathbf{E} - \tilde{\mathbf{S}}^*\mathbf{S})$ are positive. When these conditions hold, also the norms of the second column and of the two rows of the S-matrix are smaller than one.

From conditions (1) and (2) various limitations imposed by passivity on the S-matrix elements can be deduced, as, for example, the limits on an unmeasured reflection or transmission coefficient